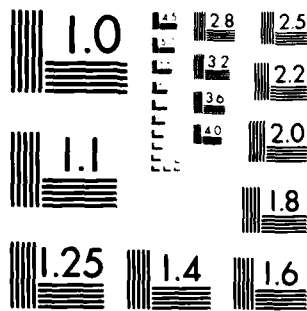


GROWTH OF FACETED CRYSTALS IN A SNOW COVER(U) COLD
REGIONS RESEARCH AND ENGINEERING LAB HANOVER NH
S C COLBECK OCT 82 CRREL-82-29

F/G 18/12 NL

END
DATE
FILMED
6-4-8
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

CRREL

REPORT 82-29



US Army Corps
of Engineers

Cold Regions Research &
Engineering Laboratory

12

AD A122792

Growth of faceted crystals in a snow cover

EQUILIBRIUM



KINETIC



FILE COPY

document has been approved
for public release and sale; its
distribution is unlimited.

DTIC
DEC 29 1982
E

TRANSITIONAL



*For conversion of SI metric units to U.S./
British customary units of measurement
consult ASTM Standard E380, Metric Prac-
tice Guide, published by the American Socie-
ty for Testing and Materials, 1916 Race St.,
Philadelphia, Pa. 19103.*

*Cover: The equilibrium form of ice grains in
a snow cover develops at slow growth
rates. The kinetic growth form devel-
ops at high growth rates; the sharp
facets disappear once the growth rate
slows and the transitional form devel-
ops.*



CRREL Report 82-29

October 1982

Growth of faceted crystals in a snow cover

Samuel C. Colbeck

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CRREL Report 82-29	2. GOVT ACCESSION NO. AD 4122	3. RECIPIENT'S CATALOG NUMBER 792
4. TITLE (and Subtitle) GROWTH OF FACETED CRYSTALS IN A SNOW COVER		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Samuel C. Colbeck		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A161102AT24 Task C/E1, Work Unit 001
11. CONTROLLING OFFICE NAME AND ADDRESS Office of the Chief of Engineers Washington, D.C. 20314		12. REPORT DATE October 1982
		13. NUMBER OF PAGES 27
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Avalanches Ice grains Crystal growth Snow Depth hoar Snow crystals Heat flux Vapor flux		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Ice grains in a snow cover with a low temperature gradient assume a well-rounded equilibrium form. However, at temperature gradients of 0.1 to 0.2°C/cm (depending somewhat on temperature and snow density), the rounded grains recrystallize into a faceted kinetic growth form. The large temperature gradient must play a decisive role in moving the vapor fast enough to sustain the rapid growth rate associated with the kinetic growth form. Once the large temperature gradient is removed, the grains recrystallize back to the equilibrium form. The recrystallization occurs in either direction without a change in bulk density. The growth of faceted crystals begins at the warmer base of the snow cover where the excess vapor pressure is largest. A transition between the overlying rounded grains moves upward in time. Faceted crystals also grow just below crusts of reduced permeability, where the increased vapor accumulation can		

DD FORM 1473
1 JAN 73

EDITION OF 1 NOV 68 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Abstract (cont'd).

sustain the excess vapor pressure needed for kinetic growth.

The heat and vapor flows are described using a model based on thermodynamic equilibrium. The temperature distribution is shown to be quasi-linear at steady state in homogeneous snow. The recrystallization of the snow is modeled using the rounded grains as sources and the faceted grains as sinks. In the future this model should be extended to account for different temperatures among the sources and sinks. <

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

PREFACE

This report was prepared by Dr. Samuel C. Colbeck, Geophysicist, Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this research was provided by DA Project 4A161102AT24/C/E1/001, *Research in Snow Mechanics*.

The manuscript of this report was technically reviewed by Dr. William St. Lawrence, Dr. Yin-Chao Yen, and Sir Charles Frank. Discussions with Sir Charles Frank, Dr. William St. Lawrence and Dr. Hans Gubler were most helpful.

The contents of this report are not to be used for advertising or promotional purposes. Citation of brand names does not constitute an official endorsement or approval of the use of such commercial products.

Acquisition For	
DTIC	<input checked="" type="checkbox"/>
DDP	<input type="checkbox"/>
DDP	<input type="checkbox"/>
DDP	<input type="checkbox"/>
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	



CONTENTS

	Page
Abstract	i
Preface	iii
Introduction	1
Vapor flow	3
Temperature profile	5
Crystal growth rates	9
Equilibrium versus kinetic growth forms of snow crystals	11
Discussion	14
Summary	16
Literature cited	17

ILLUSTRATIONS

Figure

1. Depth hoar crystals that grew in a shallow snow cover in the northeastern United States	1
2. Two photographs of seasonal snow held at a constant, subfreezing temperature in a laboratory for several years	2
3. Isopleths of equal contributions to vapor flux from temperature and radius of curvature gradients for different particle radii	5
4. The distribution of temperature in semi-infinite and 100-cm snow covers for various assumed basal temperature gradients	7
5. Ratio of vapor flux to heat flux versus temperature	8
6. Calculated distribution of the ratio of faceted crystal density to fully rounded crystal density	10
7. The excess vapor pressure is shown versus depth for various times during the re-crystallization shown in Figure 6	11
8. Two photographs of some remnants of the depth hoar crystals shown in Figure 1 after being held at -5°C for 8 months	12
9. The steady state temperature distribution below an impermeable crust for various values of the parameter n	15

GROWTH OF FACETED CRYSTALS IN A SNOW COVER

Samuel C. Colbeck

INTRODUCTION

Snow is a fascinating crystalline material that is of interest in many ways. For example, normal changes in weather patterns often cause rapid changes in the various crystal types that compose a seasonal snow cover. The fact that snow can adjust rapidly to new environmental conditions is illustrated by the rapid hardening of the outside of an igloo as vapor diffuses from the relatively warm interior to the cold exterior. In a similar fashion snow on the ground can undergo a remarkable recrystallization as vapor moves upward through the pore space away from the warmer ground. In this situation faceted crystals grow at the expense of fully rounded crystals, often leading to the development of "depth hoar," a cohesionless faceted crystal (see Fig. 1) which is sometimes responsible for avalanche release (Seligman 1936, p. 68).

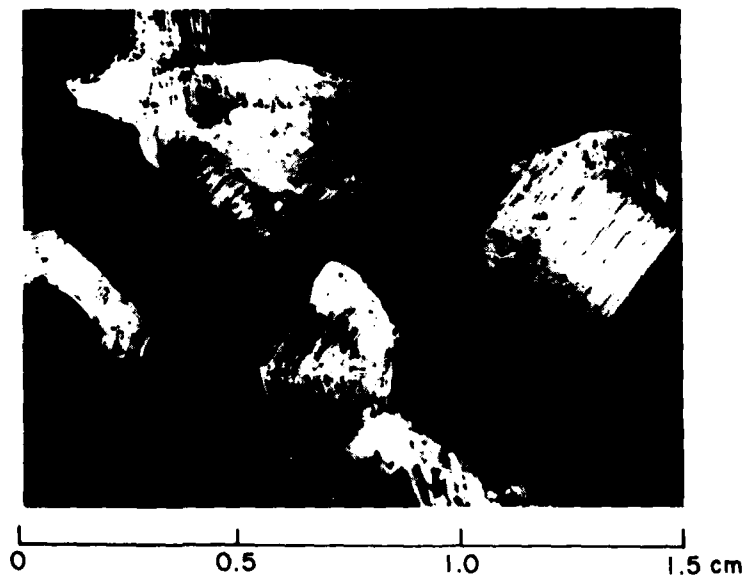
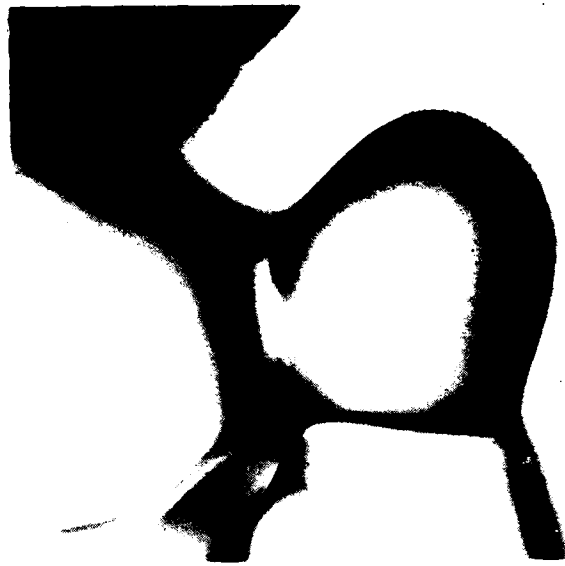
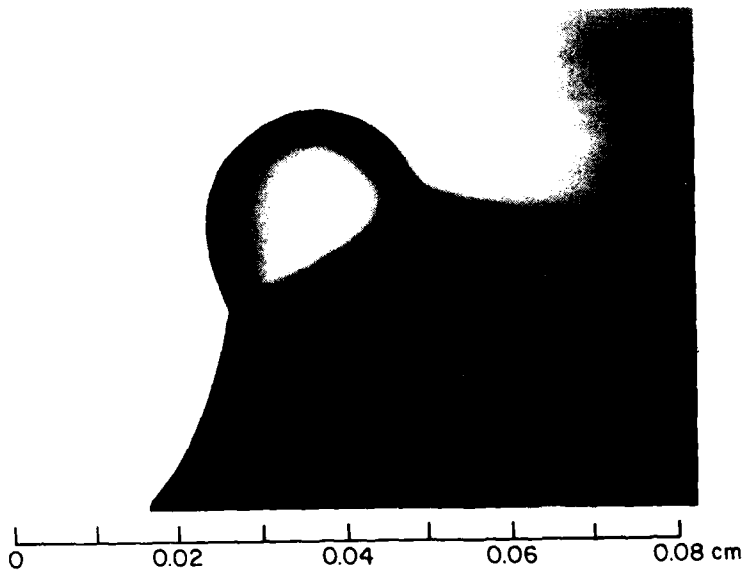


Figure 1. Depth hoar crystals that grew in a shallow snow cover (about 15 cm) in the northeastern United States. These crystals are cohesionless.



a.



b.

Figure 2. Two photographs of seasonal snow held at a constant, subfreezing temperature (no temperature gradient) in a laboratory for several years. Under these conditions the grains grow very slowly and develop a well rounded shape. The equilibrium form may be spheroidal rather than spherical and it may be faceted at lower temperatures.

Seligman (1936) referred to the early observations of Wolley (1858) and Paulcke (1932), who found both hollow and solid types of depth hoar (the hollow version is shown in Fig. 1) and reported crystal sizes as large as 1.25 in. (3.18 cm). The growth of depth hoar as well as other faceted grains was observed in the laboratory by de Quervain (1958) where the temperature and temperature gradient could be controlled over long periods of time. He found that fully rounded crystals (see Fig. 2) prevailed at low temperature gradients, while faceted crystals grew in snow with high temperature gradients (with depth hoar development at the highest gradient). The Commission on Snow and Ice (1954) classified these faceted crystals as "class d" or "depth hoar" depending on the degree of recrystallization, while Sommerfeld and LaChapelle (1970) put faceted crystals into a class they called "temperature-gradient metamorphism" because of the obvious involvement of the upward heat and vapor flows.

In addition to the importance of the seasonal snow cover per se, it is worth understanding the development of both the rounded and faceted crystals because the snow cover is a natural laboratory in which the slow processes of crystal growth at a low supersaturation can be studied. I reviewed the growth of fully rounded snow grains in the absence of an imposed temperature gradient (Colbeck 1980); the grain configuration in snow with the liquid phase present is also well understood (Colbeck 1979). However, the growth of the faceted grains and, in particular, the transition from conditions of rounded grain growth to faceted grain growth are not as well understood. Giddings and LaChapelle (1962), Yosida and Colleagues (1955), de Quervain (1963), Yen (1969), and Palm and Tveitereid (1979) have calculated the vapor movement associated with a temperature gradient in snow and, while some understanding of vapor movement in a homogeneous, infinite snow cover has been achieved, a number of important questions remain:

1. What are the conditions of temperature, temperature gradient, vapor pressure, heat flux, vapor flux and overburden pressure that favor the growth of rounded versus faceted grains? Why is there a transition between the growth regimes of rounded and faceted crystals?
2. What are the dominant mechanisms for the growth of fully rounded versus fully faceted grains?
3. Are faceted grains stable only during active growth?
4. Why do grain bonds develop in snow consisting of fully rounded grains while depth hoar is nearly cohesionless?
5. How do internal snow layers and other snow properties affect the growth of faceted grains?

VAPOR FLOW

During the period of recrystallization when faceted grains grow in a snow cover, the snow is warmed from below by the ground and cooled from above by the cold winter air. Water vapor moves upward according to

$$J = -D \frac{d\rho}{dz} + S\rho\phi \quad (1)$$

where J is the flux of water vapor, D is its diffusion coefficient in snow (depending on porosity ϕ), ρ is its density, z is the vertical coordinate, and S is the velocity of air. We assume that thermal convection is negligible, although it may be important in shallow snow covers in very cold environs. The flux can be approximated from the Clausius-Clapeyron relation as

$$J = - \frac{Dp}{RT^2} \left(\frac{L}{RT} - 1 \right) \frac{dT}{dz} \quad (2)$$

where R is the gas constant for water vapor, T is the temperature in K, L is the latent heat of sublimation, p is the vapor pressure, and dT/dz is the temperature gradient.

The equilibrium vapor pressure over a well-rounded ice surface varies with temperature and the mean radius of curvature of the ice/vapor interface. For a specified mean radius of curvature (r_0), the Clausius-Clapeyron equation relates vapor pressure and temperature,

$$p(r_0, T) = p_0 \exp \left[\frac{L}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (3)$$

where T_0 is fixed and p_0 is the vapor pressure over an ice surface with that mean radius and temperature. For a fixed temperature (T') the Kelvin equation relates vapor pressure and mean radius of curvature

$$p(r, T') = p' \exp \left(\frac{1}{\rho_i R T'} - \frac{2\sigma}{r} \right) \quad (4)$$

where p' is the vapor pressure over a flat ice surface at that temperature, σ is the interfacial energy, ρ_i is the density of ice, and r is the mean radius of curvature defined by

$$\frac{2}{r} = \frac{1}{r_1} + \frac{1}{r_2} \quad (5)$$

where r_1 and r_2 are the two principal radii of curvature of the surface (or the radii of curvature measured in any two orthogonal planes). The use of Kelvin's equation (eq 4) and the mean radius of curvature is valid only if the surface free energy is independent of orientation in ice at the temperatures considered. The evidence from snow crystals such as those shown in Figure 2 is that the equilibrium form is often sphere-like and that the surface free energy is essentially isotropic, possibly because of a disordered surface layer covering the ice that is at temperatures close to the melting point. Since this disordered layer could be much thinner at lower temperatures (Golecki and Jaccard 1978), the equilibrium form of ice crystals at lower temperatures could be the hexagonal prism suggested by Krastanow (1941). Then the equilibrium vapor pressure over a crystalline facet would have to be described by σ_j/h_j where σ_j is the surface free energy for a face of orientation j and h_j is the perpendicular distance of the Wulff center from a plane tangent to the crystal at orientation j .

Assuming that the vapor pressure is determined by the temperature and mean radius of curvature at each depth (this assumption may not hold at flow restrictions such as ice layers and the soil/snow boundary), we take

$$\frac{dp}{dz} = \frac{\partial p}{\partial T} \frac{dT}{dz} + \frac{\partial p}{\partial r} \frac{dr}{dz} \quad (6)$$

where $\partial p/\partial T$ and $\partial p/\partial r$ can be determined from eq 3 and 4 respectively. Combining eq 1, 3, 4 and 6, we find that the vapor flux is given by

$$J = - \frac{Dp}{R^2 T^2} \left(\frac{L}{T} \frac{dT}{dz} - \frac{2\sigma}{\rho_i r^2} \frac{dr}{dz} \right) \quad (7)$$

where the first term is the contribution of the temperature gradient and the second term arises from variations in the radius of curvature.

Isopleths of equal contributions to the vapor flux from the temperature and radius of curvature gradients are shown in Figure 3 for snow grains of various sizes. The radius of curvature differences between snow layers are normally quite small while temperature gradients as large as 0.5 °C/cm are not unusual (the mean radius of curvature of fully rounded grains usually falls between 10^{-3} and 10^{-1} cm). Accordingly, the vapor movement upward through a snow cover is dominated by the temperature gradient and differences in radius of curvature among particles can only be important locally. For particles of the size of snowflake branches (about 10^{-3} cm), the radius of

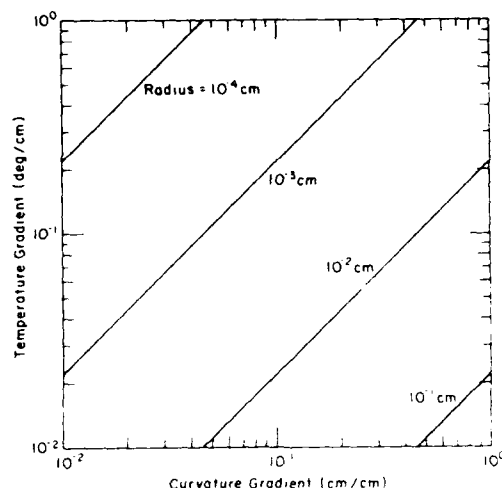


Figure 3. Isopleths of equal contributions to vapor flux from temperature ($^{\circ}\text{C}$) and radius of curvature gradients for different particle radii. In the upper left, flux is dominated by temperature gradients and in the lower right, flux is dominated by radius of curvature gradients.

curvature differences can make a substantial contribution to the vapor flux under most conditions, but these vapor pressure gradients due to curvature variations drop markedly once the initial metamorphism eliminates the original snow dendrites. In the laboratory in the absence of an imposed temperature gradient, the complete destruction of snow dendrites takes many days (de Quervain 1954, Bader 1962). However, in a seasonal snow cover, where temperature gradients are imposed by nature, we observe that the destruction of the dendrites and the growth of fully rounded grains is much faster than what occurs under isolated conditions in a laboratory. It follows that the temperature gradient term in eq 7 must be an important part of the metamorphism of snow under natural conditions, even when only fully rounded grains develop.

As I describe in more detail later, the vapor flux arising from the temperature gradient contributes significantly to the rate of metamorphism, whether fully rounded or faceted grains are the dominant form. Since the vapor flux is the rate-limiting process in the growth of rounded crystals (Colbeck 1980), the presence of a temperature gradient can greatly accelerate grain growth. For example with 10^{-2} cm grains and a temperature gradient of $0.1^{\circ}\text{C}/\text{cm}$, the temperature gradient is about 50 times more effective at moving vapor than the radius of curvature differences generally found in a snow cover.

At higher temperature gradients the growth of faceted crystals is dominant and fully rounded crystals are commonly observed to disappear. I investigate this recrystallization in several ways: first I examine heat and vapor flow in more detail and then I examine the relationship between the crystal habit and the growth kinetics.

TEMPERATURE PROFILE

Vapor flow is generally the rate-limiting process in snow metamorphism (Colbeck 1980), which explains why convection, which contributes greatly to vapor flow at very high temperature gradients, could enhance the rate of growth of depth hoar, as deduced by Trabant and Benson (1972).

The rate of increase in snow density ($\dot{\rho}_s$) due to vapor flow alone is given by the conservation of mass (other mechanisms may also contribute to snow densification; this equation describes only changes due to the gradient of vapor flow),

$$\frac{dI}{dz} + \dot{\rho}_s = 0. \quad (8)$$

From eq 1, where variations in convection are ignored and the snow cover is assumed to be homogeneous,

$$-D \frac{d^2 \rho}{dz^2} + \dot{\rho}_s = 0. \quad (9)$$

Heat flow is

$$q = -k \frac{dT}{dz} + [L + c_p (T - T_m)] I \quad (10)$$

where k is thermal conductivity due to heat conduction alone, c_p is the heat capacity of water vapor at constant pressure, and T_m is the melting temperature (normally $L \gg c_p [T - T_m]$). For the conservation of heat, in steady state,

$$\frac{dq}{dz} - L \dot{\rho}_s = 0 \quad (11)$$

where $\dot{\rho}_s$ is again the rate of change of density due only to condensation or evaporation. Combining eq 9, 10 and 11 and using $L \gg c_p (T - T_m)$, we find that the distribution of temperature and vapor density in a semi-infinite snow cover is

$$T = T_o + \frac{g}{k} z + 2 \frac{LD}{k} (\rho_o - \rho) \quad (12)$$

where ρ_o and T_o are the vapor density and temperature at the snow/ground interface and

$$g = 2L \left(D \frac{d\rho}{dz} \right)_o + k \left. \frac{dT}{dz} \right|_o. \quad (13)$$

The first term in g arises from the latent heat flux and the second term from the heat conduction across the lower boundary. The first term is zero unless the underlying soil acts as a source of water vapor. Thus g represents an important boundary condition.

The temperature and vapor density in a homogeneous snow cover without transients vary with height above the ground surface as shown in eq 12. Snow is a finely dispersed material with a large specific surface area; hence, the vapor pressure cannot deviate far from thermodynamic equilibrium as given by eq 3 and 4. Thus eq 12 shows that the temperature profile at steady state in a homogeneous snow cover must be quasi-linear; the vapor term is relatively small except at the soil/snow boundary, where the small vapor flow out of the warm soil may lead to undersaturation of the pore air. This is an important aspect of the heat and mass transfer in snow because soil, being of a lower porosity, tends to conduct heat more readily than snow while snow tends to pass water vapor more readily than soil. The resulting undersaturation at the snow/soil surface leads to the decrease in snow density reported by Giddings and LaChapelle (1962).

As stated earlier, various authors have described the general features of vapor flow in homogeneous snow without flow restrictions. It has always been assumed that the vapor pressure is given by thermodynamic equilibrium over a flat surface (Palm and Tveitereid's [1979] assumption was even more restrictive). However, since no crystal growth can occur at equilibrium, the calculated

density changes are impossible because equilibrium was assumed. Complete equilibrium requires three conditions: the vapor pressure will be given by the specified temperature and mean radius of curvature, the gradient of the vapor flux must be zero since the snow density is constant, and the gradient of the heat flux must be zero since there is no net latent heat exchange. Recrystallization, for example the simultaneous sublimation of small fully rounded crystals concurrent with the growth of faceted crystals described by Yosida and Colleagues (1955) as the "hand-to-hand delivery of water vapor," can occur under these conditions. However, it must be understood that only the average conditions and not local environments around the ice grains are being described by thermodynamic equilibrium. The rate of growth of these crystals is described later, but first we find the heat and vapor flow in a snow cover that has no vapor barriers or transients.

Assuming that thermodynamic equilibrium prevails, we see that

$$\frac{dJ}{dz} = \frac{dq}{dz} = \rho_s = 0 \quad (14)$$

and the vapor pressure is given by the Clausius-Clapeyron equation (eq 3). Then from eq 1 and 10, in the absence of air movement, the temperature distribution is given by the implicit equation (for $L \gg c_p [T - T_o]$)

$$q_o z + k(T - T_o) + \frac{LDp_o}{R} \left(\frac{e^\lambda}{T} - \frac{1}{T_o} \right) = 0 \quad (15)$$

where λ is $\frac{L}{R} \left(\frac{1}{T_o} - \frac{1}{T} \right)$.

The vapor flux is restricted by

$$J_o = - \frac{Dp_o}{R^2} \frac{L - RT_o}{T_o^3} \left. \frac{dT}{dz} \right|_o \quad (16)$$

and the vapor density is

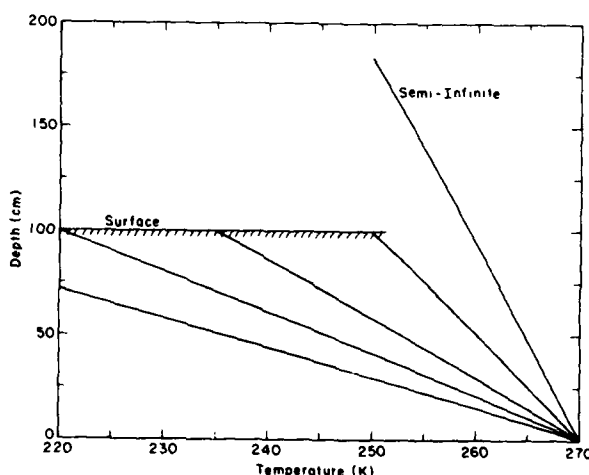


Figure 4. The distribution of temperature in semi-infinite and 100-cm snow covers for various assumed basal temperature gradients (for $k = 12,600 \text{ erg cm}^{-1} \text{ } ^\circ\text{C}^{-1} \text{ s}^{-1}$). Transients and conductivity variations have been ignored.

$$\rho = \rho_0 + \frac{p_0}{R^2} \frac{L - RT_0}{T_0^3} \frac{dT}{dz} \Big|_0 \quad (17)$$

where the subscript refers to the conditions at the soil/snow boundary. It is important that the equilibrium condition can prevail only when the vapor flux at the boundary is constrained as shown by eq 16. As shown in Figure 4, this temperature distribution is nearly linear for usual values of the parameters. The temperature profiles reported by Armstrong et al. (1974) generally confirm this quasi-linearity, although they show some variation among different layers and considerable variation near the snow surface due to radiation penetration and diurnal variations.

The ratio of vapor flux to heat flux for pure diffusion is from eq 1 and 10,

$$\frac{J}{q} = \frac{D dp/dT}{k + DLdp/dT} \quad (18)$$

Taking the gradient of vapor pressure with respect to temperature from the Clausius-Clapeyron equation (eq 3), we find that this ratio is

$$\frac{J}{q} = \frac{Dp_0(L - RT)e^\lambda}{kR^2T^3 + DLp_0(L - RT)e^\lambda} \quad (19)$$

The fact that this ratio increases at higher temperatures, as shown in Figure 5, partly explains why faceted crystals are observed to form at the base of the snow cover where it is warmer. As discussed later the kinetic growth form of the fully faceted crystals only appears when larger vapor pressures and growth rates can be sustained. Increased diffusion or thermal convection would thus tend to favor the growth of faceted crystals.

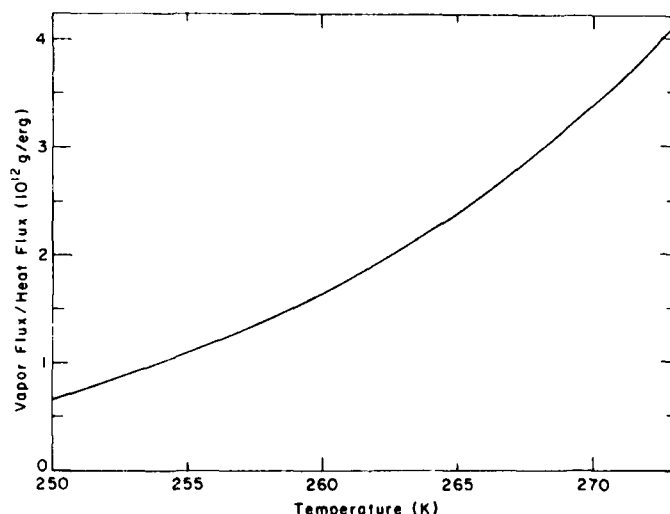


Figure 5. Ratio of vapor flux to heat flux versus temperature.

CRYSTAL GROWTH RATES

The growth of faceted crystals at the expense of the fully rounded crystals can only occur if the vapor pressure lies between that for a fully rounded crystal and that for a non-singular flat-face crystal. This vapor pressure difference could be increased if the temperature differences among the grains are increased by concentrated heat flow through the highly conductive ice grains, but this effect will have to be quantified later. We explore this recrystallization here without assigning different temperatures to the rounded and faceted crystals.

According to Wulff's theorem the equilibrium vapor pressure over a flat-faced crystal depends on the crystal size. For crystals greater than $1 \mu\text{m}$, however, the supersaturation needed for a reasonable growth rate is much greater than the supersaturation caused by a departure from the equilibrium shape (Frank 1958). Thus rapidly growing crystals are not necessarily in equilibrium.

The growth rate of a crystal increases as the supersaturation in the vapor increases. Since, without temperature differences, the supersaturation can only vary over a range of 10^{-5} between that for a flat surface and that for a rounded crystal, a linear relationship between growth rate and supersaturation is assumed, although Burton et al. (1951) suggested a parabolic relationship for dislocation-aided growth at supersaturations below 10% of the equilibrium vapor pressure. Lewis (1974b) reports that the linear relationship has been observed at low supersaturations.

The flux of molecules to the surface of a crystal is given by $c\sigma'(2\pi mk'T)^{1/2}$ where c is the condensation coefficient, σ' the supersaturation, m the molecular mass, and k' is Boltzmann's constant. The rate of increase in density of faceted crystals (ρ_f) is given by the surface area per unit volume of these crystals (approximately of $3\rho_f/r_f\rho_i$ for equant particles) times the molecular mass times the flux of molecules to the surfaces, or

$$\frac{d\rho_f}{dt} = \frac{c m [p - p(T)]}{(2\pi m k' T)^{1/2}} \frac{3 \rho_f}{\rho_i r_f} \quad (20)$$

where p is the actual vapor pressure, $p(T)$ is the vapor pressure over a flat ice surface as given by eq 3 for any specified geometry, and ρ_i is the density of ice. Likewise, the rate of growth (or evaporation) of fully rounded crystals is

$$\frac{d\rho_r}{dt} = \frac{c m [p - p(r)]}{(2\pi m k' T)^{1/2}} \frac{3 \rho_r}{\rho_i r_r} \quad (21)$$

where $p(r)$ is the equilibrium vapor pressure over a fully rounded crystal from eq 4 and r_f is the "effective radius" of a fully rounded crystal.

The increase in snow density away from impermeable boundaries over a period of weeks as measured during faceted grain growth (Armstrong 1980, Marbouty 1980) is generally very small. Accordingly, the rate of increase in the density of faceted crystals must about equal the rate of decrease in density of fully rounded crystals while the total snow density equals the sum of the densities of the two types of crystals. Therefore, combining eq 20 and 21 to eliminate p , we have

$$\int_{\rho_{f,0}}^{\rho_f} \frac{(\rho_s - \rho_f) r_f + \rho_f r_r}{\rho_f (\rho_s - \rho_f)} d\rho_f = \frac{3 c m [p(r) - p(T)]}{\rho_i (2\pi m k' T)^{1/2}} t \quad (22)$$

where the initial density of faceted crystals is $\rho_{f,0}$. Vapor pressures $p(T)$ and $p(r)$ can be obtained from eq 3 and 4, respectively, using known distributions of temperature and mean radius of curvature.

Assuming a simplified case of constant and equal crystal sizes, we see that eq 22 yields

$$\log \left[\frac{\rho_f}{\rho_f - \rho_s} \right] = \frac{3 \epsilon m [\rho(r) - \rho(T)]}{r \rho_i (2\pi m k' T)^{1/2}} r - \log \left[\frac{\rho_s - \rho_{f,0}}{\rho_{f,0}} \right] \quad (23)$$

Given the temperature profile, the size of the crystals and the initial concentration of faceted crystals, we can calculate the distribution of the density of faceted crystals with depth and time. Such a distribution is shown in Figure 6 where the ratio of faceted to fully rounded crystal densities is shown to increase rapidly near the bottom of the snow cover. This rapid increase arises because of the feedback between the crystal growth rate and the crystal surface area available for vapor deposition. Once a layer of faceted crystals builds at the base, a transition between fully rounded and faceted crystals sweeps upwards with time. This mode of formation is common in nature (Bradley et al. 1977), where it is accentuated by the layering of the snow cover and perhaps by transient effects as well. Because of the feedback mechanism between crystal growth rate and surface area, the choice of an arbitrary initial condition has little effect on the growth rate after a short period of time.

It is implicitly assumed for the moment that the vapor supply is adequate to maintain the rapid growth rates shown in Figure 6. The role of the temperature gradient in sustaining the necessary vapor flow is obvious, and it is well known that recrystallization cannot take place without a large temperature gradient. Once the density of the faceted crystals is known as a function of depth and time, the actual vapor pressure can be calculated from

$$p = \frac{\rho_f \rho(r) + \rho_s \rho(T)}{\rho_s} \quad (24)$$

which arises from eq 20 and 21 and conservation of mass. Again, this equation could be modified to account for temperature differences among the grains, a level of sophistication which will have to be added later.

The excess vapor pressure (relative to a flat ice surface at the temperature corresponding to each depth) required to drive the recrystallization shown in Figure 6 is itself shown in Figure 7. Initially, the vapor pressure is close to the equilibrium vapor pressure over curved ice grains, $p(r)$, which increases with depth because of the increasing temperature with depth. However, the vapor pressure at the base of the snow cover quickly approaches the equilibrium vapor pressure over a flat ice

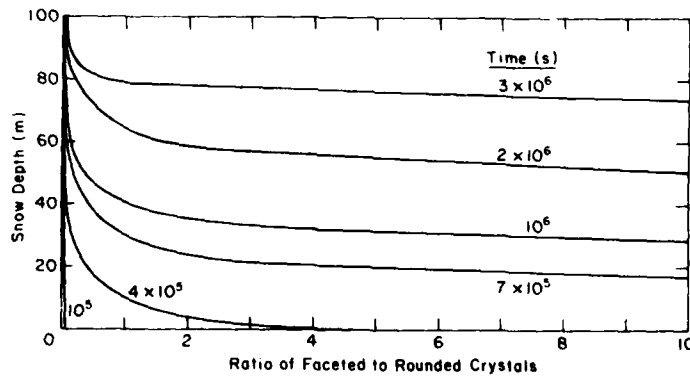


Figure 6. Calculated distribution of the ratio of faceted crystal density to fully rounded crystal density for a 100-cm snow cover with a basal temperature of 270 K, surface temperature of 235 K, crystal radius of 0.05 cm, snow density of 0.20 g/cm³, and initial density of fully faceted crystals of 0.002 g/cm³. The distribution is shown for various times and for an assumed value of 0.5 for the condensation coefficient.

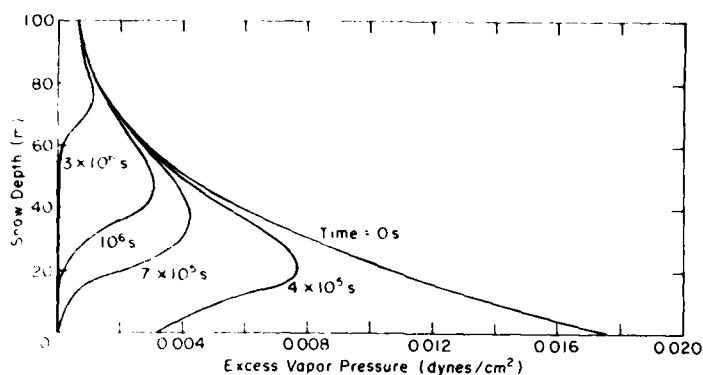


Figure 7. The excess vapor pressure (relative to a flat ice surface at the temperature corresponding to each depth) is shown versus depth for various times during the recrystallization shown in Figure 6. Temperature differences among sources and sinks would further increase the vapor pressure.

surface, $p(T)$, because of the rapid recrystallization taking place at the bottom. At least one reason for the rapid growth of the faceted grains in the lower, warmer portions of the snow cover is apparent from Figure 7: the growth rate of faceted crystals increases with the excess vapor pressure, which in turn increases with temperature. In this simplified model the level of maximum excess pressure and maximum rate of disappearance of the rounded grains increases with height as the fully rounded crystals are depleted near the bottom. Also, the surface of the snow is little affected by the rapid recrystallization occurring below. The vapor pressure and density of faceted crystals change very slowly near the upper surface; hence, the depth hoar is not evident at the surface.

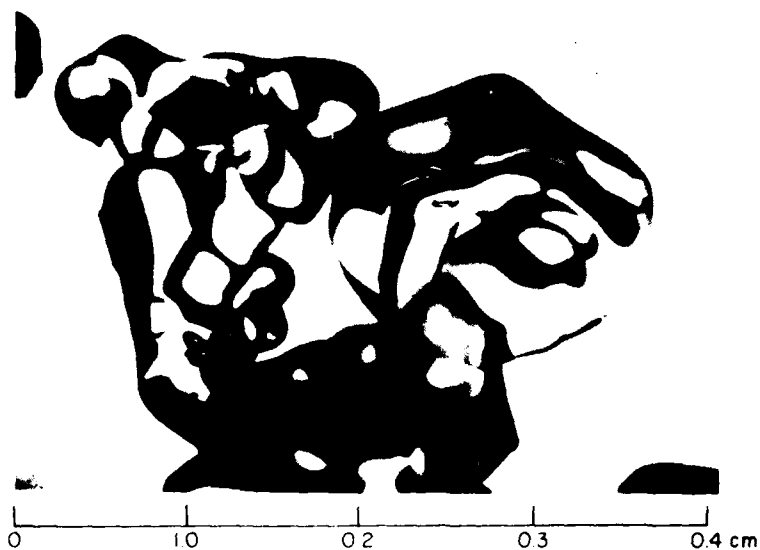
The equilibrium vapor pressure of the fully rounded crystals is higher than the prevailing vapor pressure during recrystallization, so the fully rounded grains evaporate while the faceted grains grow by vapor condensation. However, well-developed depth hoar in the laboratory occurs only in the presence of a temperature gradient of at least 0.17 to 0.25 °C/cm (Akitaya 1974, de Quervain 1958, Marbouty 1980) for reasons that are not clear. In particular, we must examine why the high pressure fully rounded crystals do not spontaneously recrystallize into lower pressure faceted crystals at any temperature gradient.

EQUILIBRIUM VERSUS KINETIC GROWTH FORMS OF SNOW CRYSTALS

The growth of individual ice crystals has been observed often in experimental apparatus in the laboratory (e.g. Nakaya 1954) at growth rates and supersaturations much larger than those shown in Figure 7. Nevertheless, Marbouty (1980) finds an approximate correspondence of the variation of crystal type with temperature between the individually grown crystals in the laboratory and crystals grown within snow at large temperature gradients. At smaller temperature gradients, faceted crystals fail to develop in the snow cover and are in fact unstable. That is, in the absence of a large temperature gradient, the faceted crystals shown in Figure 1 revert back to fully rounded crystals (as shown in transition in Fig. 8). The apparent mechanism for this rounding is the movement of molecules away from the corners because, unless the crystal is rapidly growing, the equilibrium vapor pressure over the corners of an ice crystal is very large due to the small radius of curvature (see eq 4). Hobbs and Mason (1964) and Hobbs and Radke (1967) used eq 4 to describe the growth of intergrain bonds in snow in an isolated environment. In the absence of imposed



a.



b.

Figure 8. Two photographs of some remnants of the depth hoar crystals shown in Figure 1 after being held at -5°C (no temperature gradient) for 8 months. The faceted crystal structure is unstable; hence, the snow is in the process of reverting to its stable rounded condition. Intergranular bonds develop simultaneously.

temperature gradients, the growth of fully rounded ice crystals is described by the same equation (Colbeck 1980).

The active growth of ice crystals at large supersaturations is characterized by the growth of faceted crystals, where the crystal habit is determined primarily by the temperature of growth (Nakaya 1954) and the secondary features are determined by the supersaturation (Hallet and Mason 1958, Kobayashi 1961). Lamb and Hobbs (1971) account for the variation in habit by the variation with temperature of the growth rates of the prism and basal faces. During rapid growth the diffusional limitations in the gas phase affect the crystal shape (Faktor et al. 1971, Nenow and Stoyanova 1977, Gonda 1980), a very important fact for crystal growth in the snow cover because of the presence of air in the pores of the snow. Lamb and Scott (1972) show that highly rounded crystals tend to grow in pure vapor because the vapor supply is more uniform in the absence of air. They observed slower growth rates in the presence of air. As stated earlier the slow diffusion of water vapor is the rate-limiting process in the growth of crystals in a snow cover.

Faceted ice crystals with some rounded portions can develop at temperatures close to the melting point (Pavlovskaya and Nenow 1972) and are sometimes observed in snow covers. Gilmer and Bennema (1972) explain these crystals by suggesting a critical temperature above which some crystal surfaces are "roughened" by thermal fluctuations such that growth can occur without nucleation or the screw dislocation mechanism of Burton et al. (1951). Of course at the melting point faceted crystals disappear altogether, except during very rapid growth or evaporation.

Crystal shape during rapid growth from the vapor phase is determined by the kinetics of the surface processes rather than by the shape which minimizes the surface energy (Herring 1953). At low supersaturations the growth on the low energy planes (the basal and prism faces in ice) is limited by the nucleation rate of low-energy kink sites, whereas the high energy planes (prismatic planes in ice) grow out because kink sites are plentiful (Lewis 1974a). Cahn (1960) suggests a transition from non-singular (rounded) to singular (plane) crystal surfaces by the application of a sufficient driving force for growth (i.e. a sufficient supersaturation). Hirth and Pound (1963) state that crystals which are non-singular at equilibrium may become vicinal (i.e. they exhibit a step structure in which low-index steps are separated by monomolecular risers) with increasing supersaturation. In this case, the relaxation time for the roughening of the low-index surfaces must be long, compared to the period for the addition of a monomolecular layer. In other words, the growth rate must be sufficient to overcome the tendency toward relaxation of the step structure necessary for growth.

The critical vapor pressure for the onset of "ideal" growth by spiral dislocations that intersect free surfaces as given by Hirth and Pound (1963) is

$$p/p_0 = \exp (19\sigma\Omega/\sqrt{2} x k' T) \quad (25)$$

where x is the average diffusional distance of a molecule on the surface, k' is Boltzmann's constant and Ω is the molecular volume. According to Mason et al. (1963), the mean diffusional distance for ice is about 4×10^{-4} cm; thus, the critical vapor pressure ratio is about 1.0029:1. If the evaporating rounded snow grains are at the same temperature as the growing faceted crystals and the snow grains are in the observed range of 0.01 to 0.1 cm, the critical excess vapor pressure from eq 25 exceeds the calculated excess vapor pressure by about 10^3 . This shows that the evaporating rounded crystals must be at a higher temperature than the growing faceted crystals. Thus the temperature differences among the particles due to their relative heights in the snow cover must be included in the future for a more complete description of faceted grain growth in snow.

In the absence of a large temperature gradient, I (Colbeck 1980) have shown that the diffusion of vapor is the rate-limiting process in the metamorphism of snow. Since the flux of vapor increases with the imposed temperature gradient (see eq 7 and Fig. 3), it is not surprising that a transition from fully rounded (equilibrium form) to faceted (kinetic growth form) crystals has been observed when the temperature gradient increases. From eq 7, a temperature gradient of $0.2^\circ\text{C}/\text{cm}$ increases

the vapor flux for normal size particles (0.02 cm) and radius of curvature gradients (0.1) by about 10^3 . Thus, in the presence of a large temperature gradient, the movement of vapor is not limited by the small radius of curvature difference among particles, but vapor flow from the evaporating rounded crystals to the growing faceted crystals is driven by the temperature gradient. When the temperature gradient is large enough to sustain the necessary growth rate, the kinetics of the growth form dictate the faceted crystal shape, whereas at lower temperature gradients and slower growth rates, the equilibrium shape dominates. These two distinctive forms have been classified as "temperature-gradient" and "equi-temperature," respectively, although the temperature gradient largely determines the growth rate in both cases. One is a kinetic growth form whereas the other is an equilibrium form.

DISCUSSION

Given the small excess vapor pressure at which the snow cover recrystallizes into faceted grains (probably less than 1% supersaturation), it does not seem possible that these grains are nucleated either homogeneously or heterogeneously by impurities (at 170 K the critical supersaturation for a large nucleation rate for water vapor is about four times the equilibrium vapor pressure over a flat surface [Hirth and Pound 1963, p. 31]). The faceted crystals grow on fully rounded ice particles; possibly, the relatively strain free and the larger particles serve as sites for faceted grain growth while the grains carrying the maximum load of overburden and the smallest grains evaporate due to their higher chemical potentials. Of most importance, the grains in a chain that conducts away more than the average amount of heat would be at a different temperature than the neighboring grains. Once the fully rounded grains have completely disappeared, the faceted grains can continue to grow by the evaporation of the stressed grains now carrying the overburden pressure or by the evaporation of grains in the most effective heat paths. If the increased chemical potential due to radius of curvature was important, grain growth could slow markedly once the fully rounded crystals have disappeared because the increase in chemical potential due to radius of curvature is large compared to the increase in chemical potential due to an elastic strain. For particles larger than 10^{-5} cm, Cabrera (1964) says that stresses are not important. This explains why de Quervain (1958) observed very little influence of overburden pressure during faceted crystal growth. However, it is likely that the radius of curvature effect is much less than the effect of preferential heat flow paths at temperature gradients of $0.2^\circ\text{C}/\text{cm}$.

So far we have assumed that the snow cover is homogeneous, whereas snow covers in nature are almost always layered. Snow covers are deposited by discrete snow storms; hence, the layers are often separated by "crusts" of fine-grained, higher density snow. These crusts have a relatively low permeability and the vapor pressure tends to increase beneath them. With the vapor pressure increased locally, faceted grains are often observed to form just below crusts in spite of the lower snow layers being warmer. Of course this vapor deposition reduces the permeability of the crust even further as the crusts densify from within the snow cover. A similar phenomenon was observed in the laboratory by de Quervain (1958) when he placed foil sheets in a snow sample and observed vapor condensation on the warm side of the sheet. On the cold side of a vapor barrier (e.g. immediately above the ground surface), a decline in snow density is often observed and in extreme conditions cavities may even form (Giddings and LaChapelle 1962).

The temperature distribution below an impermeable crust can be calculated by combining eq 1, 9, 10 and 11 with a flux distribution of the form that deposits condensate just beneath the crust,

$$J = J_0 (1 - z/H)^{1/n} \quad (26)$$

where J_0 is the flux at the base of the snow cover, n is an arbitrary constant greater than unity, and the height of the crust above the origin is H . Assuming that the vapor pressure is closely

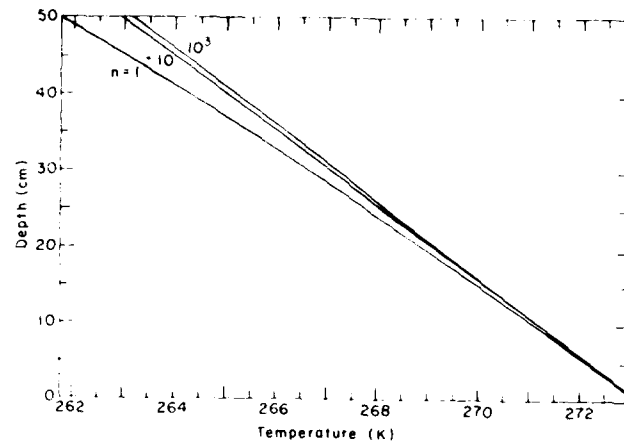


Figure 9. The steady state temperature distribution below an impermeable crust for various values of the parameter n . The temperature distribution above an impermeable crust or soil would have the opposite curvature.

approximated by the equilibrium vapor pressure (eq 3) and taking $L \gg c_p (T - T_m)$, we find that the temperature distribution is

$$T = T_o + \frac{dT}{dz} \Big|_o z - \frac{2LJ_o}{k} \left[z - \frac{nH}{1+n} + \frac{nH}{1+n} (1 - z/H)^{\frac{1+n}{n}} \right] \quad (27)$$

which is a slightly nonlinear distribution, as shown in Figure 9. The nonlinearity would increase with thermal convection, which was ignored in this derivation. If we assume an increasing vapor flux above the soil surface or above a crust in the snow, we would find a temperature distribution with a curvature of the opposite sense to that shown in Figure 9.

Several crusts are likely to develop in any particular snow cover; hence, the temperature distribution should be scalloped due to the increased deposition near the base of each crust. The quasi-linear temperature distribution of eq 15 should describe the average temperature profile, although the profile within each snow layer will have the shape of eq 27. According to eq 27 the temperature gradient reaches its largest negative value just below each crust, where the condensation rate and latent heat release are greatest.

As long as the critical temperature gradient maintains the rapid vapor flow required to achieve the kinetic growth form, snow will undergo continuous recrystallization. Marbouty (1980) found a linear increase in crystal size with time, and very large crystals (3 cm) can be observed in nature when steep temperature gradients are maintained for months. The effects of increasing crystal size with time were not included in eq 22 or Figures 6 and 7. Equation 20 suggests that the rate of density increase of the faceted crystals will decrease as their size increases; hence, the transition between the layers of fully rounded and faceted crystals would not be as sharp as suggested by Figure 6 unless the crusts, which normally exist in a snow cover, have a large effect.

It is unlikely that the average size of a fully rounded crystal would decrease in a linear fashion since the vapor pressure over a particle increases rapidly with decreasing radius (see eq 4) and the rate of evaporation varies as the vapor pressure difference divided by the radius (see eq 21). This driving mechanism for grain evaporation includes a positive feedback between grain size and the rate of change of grain size, which leads to an accelerating rate of grain shrinkage. Typically, a fully rounded grain would decrease slowly in size at first but then disappear quickly after reaching a certain critical size. This mode of sublimation of the rounded grains would have little effect on

the growth of the faceted grains, since the average size of the rounded grains would vary little with time despite the decreasing density of the rounded grains.

At low supersaturations and growth rates, crystal growth follows the kinetic growth laws for liquids. At high supersaturations and growth rates, crystals grow by the diffusion of surface-absorbed molecules to steps that propagate across the crystalline faces. At the higher growth rates, small quantities of organic vapors modify the habit of ice crystals (Hallet and Mason 1958), presumably because they influence the surface diffusion process. LaChapelle (1966) mentioned the potential use of organic compounds to prevent the growth of depth hoar in mountain snowpacks but much work remains to be done before a reliable method is developed. Elimination of the depth hoar would reduce avalanches because one characteristic of depth hoar is the lack of cohesion among the ice particles. Part of the fragile nature of depth hoar may be due to the large mechanical advantage achieved with stressed large grains; however, the large, cohesionless grains of fully developed depth hoar quickly grow intergranular bonds and strength once the temperature gradient is removed. Interparticle bonds grow between fully rounded grains by vapor diffusion due to differences in radius of curvature (Hobbs and Mason 1964) as dictated by the principles of the equilibrium form. However, the active growth of fully faceted crystals is controlled by the kinetic processes and concave grain bonds do not develop because they are an equilibrium form. Also, since depth hoar develops rapidly, there is not sufficient time for much sintering to occur. Once the temperature gradient is removed and the faceted crystals revert to the fully rounded equilibrium form, grain bonds develop while the facets disappear (see Fig. 8).

SUMMARY

Falling snow crystals are generally faceted with a crystal habit that depends primarily on the temperature of growth. Upon reaching the ground, the faceted crystals lose their angularity and assume the fully rounded equilibrium form. Initially, the snow crystals become rounded and bonded together but can recrystallize into another faceted form if the snow cover experiences a critical temperature gradient (0.1 to 0.2 °C/cm). Depth hoar, the most advanced case of the faceted form, is often responsible for avalanche release because of its lack of cohesion.

The kinetic growth form (or faceted crystals) prevails during rapid crystal growth at large supersaturations, whereas the equilibrium form (or fully rounded, well-bonded crystals) prevails during the slow growth at very low supersaturations. At very small temperature gradients, the growth rate is limited by the slow diffusion of water vapor from the evaporating smaller crystals to the growing larger crystals. In this environment of restricted vapor diffusion through air, the equilibrium form prevails. Above the critical temperature gradient, the movement of water vapor is greatly increased by the vapor pressure gradient associated with the large temperature gradient. Vapor fluxes due to the imposed temperature gradient are many times larger than those due to differences in radius of curvature; hence, with a large temperature gradient, it is possible to sustain the rapid kinetic growth rate which leads to the growth of the faceted crystals.

The growth of the faceted crystals typically starts at the base of the snow cover where the excess vapor pressures are higher because the temperature and the rate of vapor movement are highest there. Once the faceted crystals begin growing, their rate of growth increases as their total surface area increases. There may be an additional effect due to heat piping through the lengthening grains. A transition between the overlying layer of fully rounded and the underlying layer of fully faceted crystals is seen to move upward as the snow cover recrystallizes. The level of maximum rate of evaporation of the rounded crystals moves upward with time as these source crystals are depleted at lower levels. When a less permeable crust develops within a snow cover, a distinct layer of faceted grains may develop above the transition level shown in Figures 6 and 7 because of restricted vapor flow and higher excess vapor pressure beneath the crust.

The steady state temperature profiles given by eq 12 suggest a quasi-linear temperature profile away from the upper surface regardless of flow restrictions or crystal growth. Figure 9 supports this conclusion, even in the presence of an impermeable crust. Apparently, the steady state temperature profile in a snow cover is approximately linear as long as large variations in thermal conductivity do not occur among layers. This shows that the common practice of equating the actual vapor pressure and the equilibrium vapor pressure given by the Clausius-Clapeyron equation (eq 3) is reasonable if only the temperature profile is desired. However, if density changes or the onset of faceted crystal growth is being investigated, a more accurate estimate of the vapor pressure is required.

Snowflakes begin as faceted crystals nucleated in the atmosphere. Once they fall to earth, they can undergo many recrystallizations in response to changes in the snow cover's temperature gradient and liquid water content. Regardless of where they fall on earth, they always end their life as well-rounded grains in multilayer glacial snow or as well-rounded grains in a melting snow cover. The path they take from snowflakes to glacial ice or meltwater involves many complicated processes. The recrystallization of fully rounded to faceted grains in a snow cover is one of the most interesting of these processes.

LITERATURE CITED

- Akitaya, E. (1974) Studies of depth hoar. *Low Temperature Science*, vol. A, no. 26, p. 1-67.
- Armstrong, R.L. (1980) An analysis of compressive strain in adjacent temperature-gradient and equi-temperature layers in a natural snow cover. *Journal of Glaciology*, vol. 26, no. 94, p. 283-89.
- Armstrong, R.L., E.R. LaChappelle, M.J. Bouis and J.D. Ives (1974). *Development of Methodology for Evaluation and Prediction of Avalanche Hazard in the San Juan Mountain Area of Southwestern Colorado*. Occasional Paper no. 13. University of Colorado, Boulder: Institute of Arctic and Alpine Research.
- Bader, H. (1962) The physics and mechanics of snow as a material. U.S. Army Cold Regions Research and Engineering Laboratory. Cold Regions Science and Engineering Monograph II B.
- Bradley, C.C., R.L. Brown, and T.R. Williams (1977) Gradient metamorphism, zonal weakening of the snow-pack and avalanche initiation. *Journal of Glaciology*, vol. 19, no. 81, p. 335-42.
- Burton, W.K., N. Cabrera and F.C. Frank (1951) The growth of crystals and the equilibrium structure of their surfaces. *Philosophical Transactions of the Royal Society of London*, vol. A243, p. 299-358.
- Cabrera, N. (1964) The equilibrium of crystal surfaces. *Surface Science*, vol. 2, p. 320-45.
- Cahn, J.W. (1960) Theory of crystal growth and interface motion in crystalline materials. *Acta Metallurgica*, vol. 8, p. 554-62.
- Colbeck, S.C. (1979) Grain clusters in wet snow. *Journal of Colloid and Interface Science*, vol. 72, no. 3, p. 371-84.
- Colbeck, S.C. (1980) Thermodynamics of snow metamorphism due to variations in curvature. *Journal of Glaciology*, vol. 26, no. 94, p. 291-301.
- Commission on Snow and Ice (1954) *The International Classification of Snow*. Technical Memorandum No. 31. Ottawa: Association of the Commission on Soil and Snow Mechanics.
- de Quervain, M. (1954) Schnee als kristallines Aggregat. *Experimentia*, vol. 1, p. 207-12.
- de Quervain, M. (1958) *On Metamorphism and Hardening of Snow Under Constant Pressure and Temperature Gradient*. Gentbrugge, Belgium: International Association of Scientific Hydrology. Publication No. 46, p. 225-39.
- de Quervain, M. (1963) On the metamorphism of snow. In *Ice and Snow, Properties, Processes and Applications* (W.D. Kingery, Ed.). Cambridge, Massachusetts: MIT Press, p. 377-390.

- Faktor, M.M., I. Garrett and R. Heckingbottom (1971) Diffusional limitations in gas phase growth of crystals. *Journal of Crystal Growth*, vol. 9, p. 3-11.
- Frank, F.C. (1958) Introductory lecture. In *Growth and Perfection of Crystals* (R.H. Doremus, B.W. Roberts and D. Turnbull, Ed.). New York: John Wiley, p. 3-10.
- Giddings, J.C. and E. LaChapelle (1962) The formation rate of depth hoar. *Journal of Geophysical Research*, vol. 67, no. 6, p. 2377-83.
- Gilmer, G.H. and P. Bennema (1972) Simulation of crystal growth with surface diffusion. *Journal of Applied Physics*, vol. 43, no. 4, p. 1347-60.
- Golecki, I. and C. Jaccard (1978) Intrinsic surface disorder in ice near the melting point. *Journal Phys. C: Solid State Phys.*, vol. 11, p. 4229-37.
- Gonda, T. (1980) The influence of the diffusion of vapor and heat on the morphology of ice crystals grown from the vapor. *Journal of Crystal Growth*, vol. 49, no. 1, p. 173-81.
- Hallet, J. and B.J. Mason (1958) The influence of temperature and supersaturation on the habit of ice crystals grown from the vapor. *Proceedings of the Royal Society of London, Ser. A*, vol. 247, p. 440-53.
- Herring, C. (1953) The use of classical macroscopic concepts in surface-energy problems. In *Structure and Properties of Solid Surfaces* (R. Gomer and C.S. Smith, Ed.). Chicago: University of Chicago Press, p. 5-81.
- Hirth, J.P. and G.M. Pound (1963) *Condensation and Evaporation*. New York: Macmillan.
- Hobbs, P.V. and B.J. Mason (1964) The sintering and adhesion of ice. *Philosophical Magazine*, vol. 9, no. 98, p. 181-97.
- Hobbs, P.V. and L.F. Radke (1967) The role of volume diffusion in the metamorphism of snow. *Journal of Glaciology*, vol. 6, no. 48, p. 879-91.
- Kobayashi, T. (1961) The growth of snow crystals at low supersaturations. *Philosophical Magazine*, vol. 6, p. 1363-70.
- Krastanow, L. (1941) *Metallurgia Zeitschrift*, vol. 58, p. 37-45.
- LaChapelle, E.R. (1966) The control of snow avalanches. *Scientific American*, vol. 214, no. 2, p. 92-101.
- Lamb, D. and P.V. Hobbs (1971) Growth rate and habits of ice crystals grown from the vapor phase. *Journal of Atmospheric Sciences*, vol. 28, no. 8, p. 1506-09.
- Lamb, D. and W.D. Scott (1972) Linear growth rates of ice crystals grown from the vapor phase. *Journal of Crystal Growth*, vol. 12, p. 21-31.
- Lewis, B. (1974a) The growth of crystals from low supersaturation. I. Theory. *Journal of Crystal Growth*, vol. 21, p. 29-39.
- Lewis, B. (1974b) The growth of crystals from low supersaturation. II. Comparison with experiment. *Journal of Crystal Growth*, vol. 21, p. 40-50.
- Marbouty, D. (1980) An experimental study of temperature gradient metamorphism. *Journal of Glaciology*, vol. 26, no. 94, p. 303-12.
- Mason, B.J., G.W. Bryant and A.P. Van de Herval (1963) The growth habits and surface structure of ice crystals. *Philosophical Magazine*, vol. 8, p. 505-26.
- Nakaya, U. (1954) *Snow Crystals*. Cambridge, Massachusetts: Harvard University Press.
- Nenow, D. and V. Stoyanova (1977) On the formation of ice dendrites from the vapor phase. *Journal of Crystal Growth*, vol. 41, no. 1, p. 73-76.
- Palm, E. and M. Tveitereid (1979) On heat and mass flux through dry snow. *Journal of Geophysical Research*, vol. 84, no. C2, p. 745-49.
- Paulcke, W. (1932) *Der Bergsteiger*, vol. 6, p. 340.
- Pavlovskaya, A. and D. Nenow (1972) Les surfaces non-singulières sur la forme d'équilibre de naphthalène. *Journal of Crystal Growth*, vol. 12, p. 9-12.
- Seligman, G. (1936) *Snow Structure and Ski Fields*. London: Macmillan.
- Sommerfeld, R.A. and E. LaChapelle (1970) The classification of snow metamorphism. *Journal of Glaciology*, vol. 9, no. 55, p. 3-17.

- Trabant, D. and C. Benson (1972) *Field experiments on the development of depth hoar*. Memorandum 135. Geological Society of America, p. 309-22.
- Wolley, J. (1858) *Report of the British Association*. Part II, p. 40-41.
- Yen, Y.C. (1969) Recent studies in snow properties. In *Advances in Hydrosience* (V.T. Chow, Ed.), vol. 5. New York: Academic Press, p. 173-214.
- Yosida, Z. and Colleagues (1955) Physical studies of deposited snow. I. Thermal properties. *Low Temperature Science*, vol. 7, p. 19-74.

A facsimile catalog card in Library of Congress MARC format is reproduced below.

Colbeck, Samuel C.

Growth of faceted crystals in a snow cover / by Samuel C. Colbeck. Hanover, N.H.: U.S. Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1982.

iv, 27 p., illus.; 28 cm. (CRREL Report 82-29.)

Prepared for Office of the Chief of Engineers by Corps of Engineers, U.S. Army Cold Regions Research and Engineering Laboratory under DA Project 4A161102 AT24.

Bibliography: p. 17.

1. Avalanches. 2. Crystal growth. 3. Depth hoar. 4. Heat flux. 5. Ice grains. 6. Snow. 7. Snow crystals. 8. Vapor flux.

(See next card)

Colbeck, Samuel C.

Growth of faceted crystals...
1982

(Card 2)

I. United States. Army. Corps of Engineers. II. Army Cold Regions Research and Engineering Laboratory, Hanover, N.H. III. Series: CRREL Report 82-29.

**DAI
ILMI**